WESTMINSTER SCHOOL
THE CHALLENGE 2015

MATHEMATICS II

Tuesday 28th April 2015

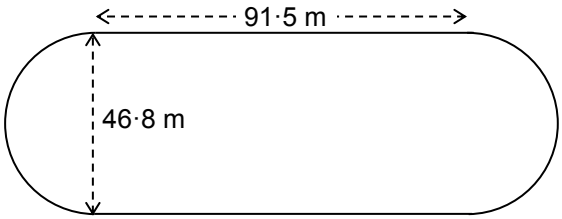
Time allowed: 1 hour 30 minutes

You will need a calculator for this paper.

All your working should be clearly shown.

You should attempt all the questions.

1 Harry runs once around the track shown below, which consists of two straight sections and two semi-circular parts, in 63·4 seconds.



Harry runs at an average speed of 4·9 metres per second on the semi-circular sections of the track. What is his average speed on the straight sections of the track?

2 **a** Multiply xy by $\frac{x}{y}$, and simplify your answer.

b Make h the subject of the formula $a^2-(a+b)h=b^2$.

c Solve the simultaneous equations

$6y-10x=19$

$6x+15y=1$.

d The linear equation

$$\frac{3x-1}{4}-\frac{x-2}{a}=1$$

has solution $x=\frac{7}{5}$. Find the value of a.

3 The four calculations below might be:

- correct only if brackets are added appropriately;
- correct without any brackets needed;
- incorrect, regardless of how brackets are placed.

Write out each of the four calculations and either add the necessary brackets or write "no brackets needed" or "not possible" after it.

i $13-3\times6\div2=4$

ii $5-2\div6\div3=4$

iii $8-6\div2\div3=4$

iv $4-2\div3\div6=4$

4 Two twenty-four hour clocks run at different (but constant) rates. Clock A always shows the correct time. On Monday, when clock A shows 08:40, clock B shows 08:50 and when clock A shows 10:10, clock B shows 10:35.

a On Monday, when clock B shows 16:25, what is the correct time?

b At what time on Monday morning did clock B show the correct time?

c At what time, and on what day, does clock B next show the correct time?

5 Josh is doing 40 minutes of exercises. He does press-ups, star-jumps, sit-ups and squats.
He does star-jumps for twice as many minutes as he does press-ups;
he does sit-ups for seven more minutes than he does press-ups;
he does squats for the remainder of the 40 minutes.

a If he does x minutes of press-ups, how many minutes of squats does he do?

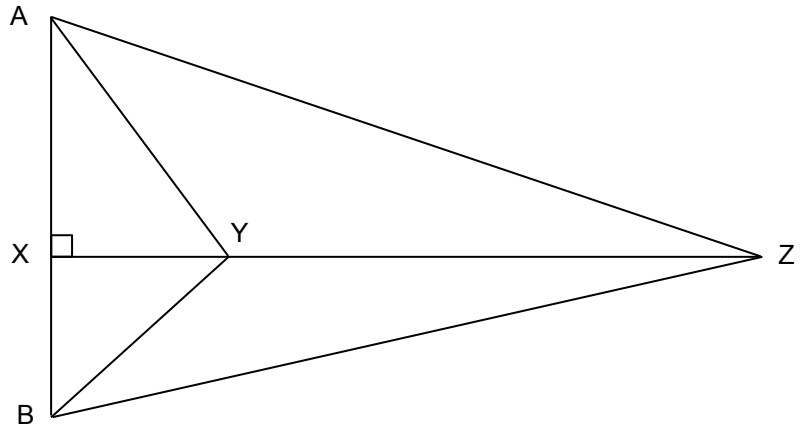
Each minute of press-ups burns 7 calories; each minute of star jumps burns 4 calories; each minute of sit-ups burns 6 calories and each minute of squats burns 8 calories

b Write an expression in terms of x for the total number of calories that Josh burns.

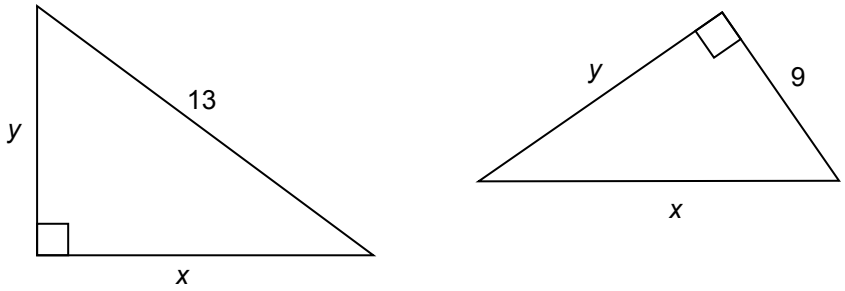
In fact, Josh burns 240 calories.

c How many minutes of press-ups did he do?

6 In the diagram AXB and XYZ are straight lines and angle AXY is a right angle.
Distance AX = 8 cm and distance XB = 5 cm.
The area of triangle AYZ is 68 cm^2.
What is the area of triangle BYZ?



7 The diagram shows two right angled triangles.
Find *x* and *y*, given that they represent the same lengths in both triangles.



8 **In this question, you should round your answers to give whole numbers of votes.**

In the 2010 election for Borsetshire East, the only candidates were from the Conservatives, the Liberal Democrats and the Labour Party. In 2010 27,487 votes were cast for the Conservative candidate, but he received 23% fewer votes in the 2015 election.

a How many votes did the Conservative candidate receive in 2015?

In 2015, 9,370 votes were cast for the Liberal Democrat candidate: this was 37% fewer than in the 2010 election.

b How many votes did the Liberal Democrat candidate receive in 2010?

The Labour candidate received 7846 votes in 2010 and 6826 votes in 2015.

c What was the percentage decrease in her vote?

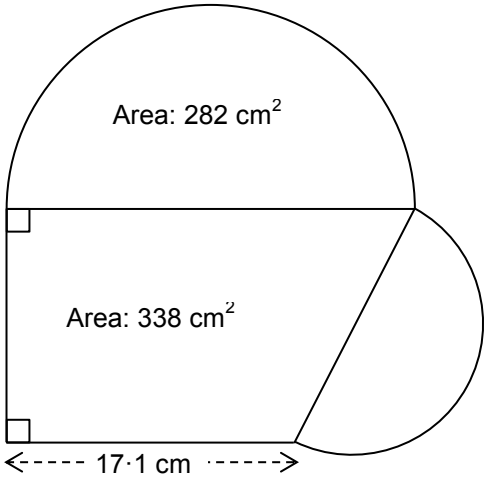
In the 2015 election, there was a fourth candidate.

d If the total number of electors voting in 2015 was 17% higher than in 2010, how many votes did this fourth candidate receive?

9 On a Scottish moor, the population of grouse fell by 32% one year, and rose by the same percentage in each of the three subsequent years. The population of grouse was then still 7% below the level it was at before the fall. By what percentage did it rise in each of the three years?

10 William has been set too many Latin tests this term, all of them out of 100, but he has done quite well. In fact, even if he had scored only 17 in his last test, his average over the term would have been 80. Actually, he manages 92 in the last test and his average over the term is 85.
Let *T* be the total of William's marks before the last test, and *n* be the number of tests he takes altogether. Use the information given to write two equations in *T* and *n*, and solve them to find how many tests William sat this term.

11 The diagram shows a trapezium and two semicircles. The areas of one of the semicircles and of the trapezium are marked on the diagram, together with the length of one side of the trapezium. Two of the angles of the trapezium, as indicated, are right angles. What is the area of the other semicircle?



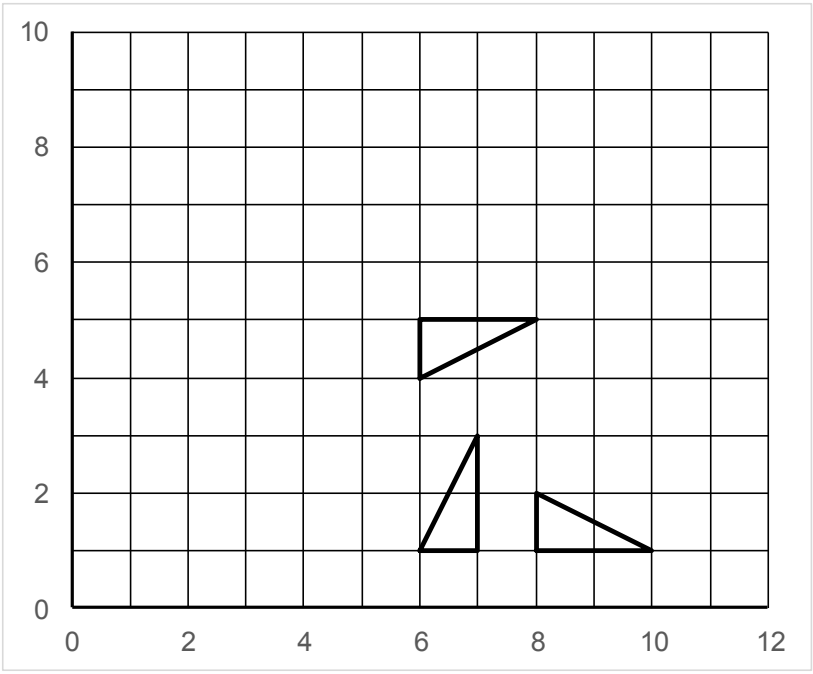
12 A teacher works out the average mark of the ten boys in his class as 68, rounded to the nearest whole number. Later he finds that he wrote down one of the marks as 87, when it should have been 78. He recalculates the average mark and finds that it is still 68, rounded to the nearest whole number. Given that each boy scored a whole number of marks, what is the exact average mark of the class?

13 A triangle **T**, which is *not* shown on the diagram below, has been

- reflected in the line $y = x$;
- rotated through 90° clockwise about the point (4, 2);
- translated by two units right and five units down;

to produce the three triangles shown on the diagram.

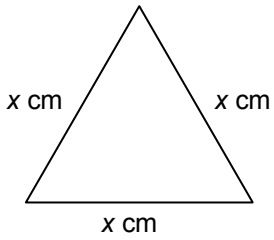
Work out where triangle **T** is and give the co-ordinates of its three vertices.
Explain clearly the method you used to decide on your answer.



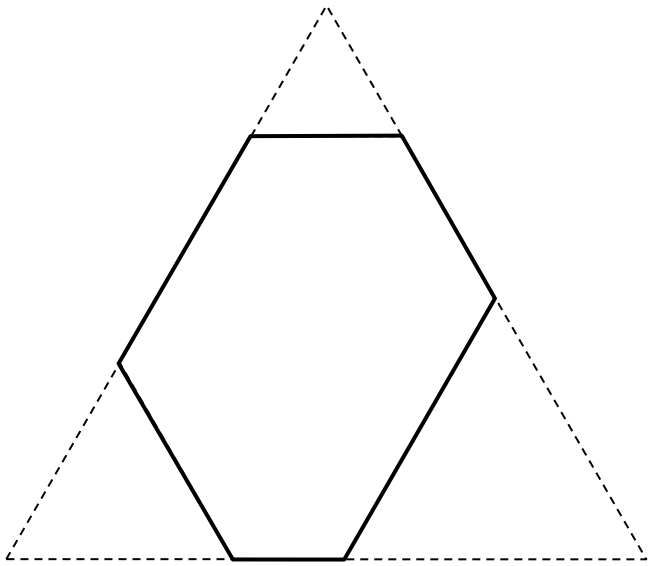
TURN OVER

14 **a** The diagram below shows an equilateral triangle with side length x cm.

Show that it has area $\frac{\sqrt{3}}{4}x^2$ cm^2.



b The diagram below shows a large equilateral triangle from which three smaller equilateral triangles, with side lengths 2 cm, 3 cm and 4 cm have been cut away.



Given that the area of the remaining hexagon is 18 cm^2, find the side length of the original triangle.